

Chapter 5

A General Discrete-to-Continuum Neutrosophic Self-Reference Stability and Fixed Points of Logical Paradox

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ABSTRACT

This chapter introduces the Neutrosophic Self-Reference Stability Theorem and studies paradox as a dynamical process inside neutrosophic truth space. Instead of producing logical collapse, self-referential sentences may converge to stable neutrosophic valuations (T, I, F). The framework connects liar paradox dynamics, Gödel self-reference, and contradictions across science, technology, administration, decision-making, literature, and art. The concept is also discussed in relation to the Infinitesimally Punctured Wave (IPW) inspired by wave-particle duality.

Keywords: neutrosophic logic; paradox dynamics; Gödel sentence; liar paradox; contradictions; stability theorem

INTRODUCTION

Neutrosophic logic [1] extends the classical [2, 3] and fuzzy [4] logic. Contradictions [5, 6] appear in many areas of knowledge. Classical logic [2, 3] treats contradictions as destructive. Neutrosophic logic [7] provides a framework in which statements may simultaneously contain degrees of truth, indeterminacy, and falsehood.

$\text{Val}(S) = (T, I, F)$, where $0 \leq T \leq 1, 0 \leq I \leq 1, 0 \leq F \leq 1$.

Classical Logical Paradoxes

Example: Liar paradox [8]

"This statement is false."

Binary evaluation produces oscillation: $\text{true} \rightarrow \text{false} \rightarrow \text{true} \rightarrow \text{false}$

This instability motivates richer truth models.

Gödel Self-Reference

Gödel [9] constructed a sentence satisfying $G \leftrightarrow \neg \text{Prov}(G)$

meaning "G is not provable".

Instead of binary valuation we may assign $\text{Val}(G) = (T_G, I_G, F_G)$.

Self-Reference as a Dynamical System

Let $X_n = (T_n, I_n, F_n)$ represent the truth state after n evaluations.

Define the transformation $X_{n+1} = f(X_n)$, where $f : [0,1]^3 \rightarrow [0,1]^3$.

Neutrosophic Self-Reference Stability Theorem

Let $N = [0,1] \times [0,1] \times [0,1]$.

If the evaluation operator $f : N \rightarrow N$ is continuous, then there exists a fixed point $X^* = (T^*, I^*, F^*)$ such that $f(X^*) = X^*$.

If extreme classical states are unstable then,

$$0 < T^* < 1$$

$$0 < I^* < 1$$

$$0 < F^* < 1.$$

Neutrosophic truth cube is shown in figure 1.

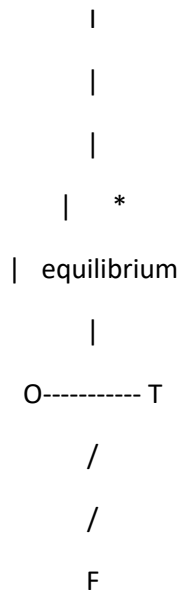


Figure 1. Neutrosophic Truth Cube (schematic)

Example Dynamical Evolution

Example iteration

$$X_0 = (1,0,0)$$

$$X_1 = (0.6,0.1,0.3)$$

$$X_2 = (0.4,0.3,0.3)$$

$$X_3 \rightarrow (0.33,0.34,0.33)$$

This approaches a stable neutrosophic equilibrium.

Catalog of Contradictions [4] Across Domains

Science: Wave–particle duality: light behaves as wave and particle.

Technology: Engineering trade-offs: speed vs accuracy.

Administration: Policy conflicts: efficiency vs social benefit.

Decision-Making: Evidence may simultaneously support and contradict a hypothesis.

Literature: Unreliable narrators create contradictory perspectives.

Art: Abstract vs representational tension in modern art.

Relation with Infinitesimally Punctured Wave

Wave-particle duality inspired the Infinitesimally Punctured Wave idea.

Logical analogy:

wave structure $\rightarrow$ theory

punctures $\rightarrow$ contradictions

Neutrosophic logic models these punctures without collapse.

Contradictory-Theory Stability Principle

Systems containing contradictory theories may stabilize at interior neutrosophic states rather than collapsing into inconsistency.

Such states correspond to equilibrium points (T^*, I^*, F^*) .

Future Research

Possible directions:

- computational simulation of paradox dynamics
- neutrosophic models of scientific theory conflict
- connections with quantum logic
- applications in decision science and governance

Conclusion

Neutrosophic logic [1] transforms paradox from contradiction into stable logical structure. Self-reference may converge to interior truth states rather than oscillating between true and false.

References

- [1] Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic*. American Research Press.
- [2] Shapiro, S., & Kouri Kissel, T. (2022). Classical logic. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Fall 2022 ed.). Stanford University.
<https://plato.stanford.edu/entries/logic-classical/>
- [3] Copi, I. M., Cohen, C., & McMahon, K. (2016). *Introduction to logic* (14th ed.). Pearson.
- [4] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.
[https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [5] Priest, G. (2006). *In Contradiction* (2nd ed.). Oxford University Press.
- [6] Horn, L. R. (2025). Contradiction. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy* (Spring 2025 ed.). Metaphysics Research Lab, Stanford University.
<https://plato.stanford.edu/archives/spr2025/entries/contradiction/>
- [7] Smarandache, F. (2003). *A unifying field in logics: Neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability* (3rd ed.). American Research Press.
- [8] Beall, J. C., Glanzberg, M., & Ripley, E. (2025). Liar paradox. In E. N. Zalta & U. Nodelman (Eds.), *Stanford encyclopedia of philosophy* (Fall 2025 ed.). Metaphysics Research Lab, Stanford University.
<https://plato.stanford.edu/archives/fall2025/entries/liar-paradox/>
- [9] Gödel, K. (1931). Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I [On formally undecidable propositions of Principia Mathematica and related systems I]. *Monatshefte für Mathematik und Physik*, 38, 173–198. <https://doi.org/10.1007/BF01700692>